Section 4.3

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric function of the angle θ are defined as follows.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

$\text{opp} =$ the length of the side opposite θ, $\text{adj} =$ the length of the side adjacent to θ, $\text{hyp} =$ the length of the hypotenuse.

Fundamental Trigonometric Identities

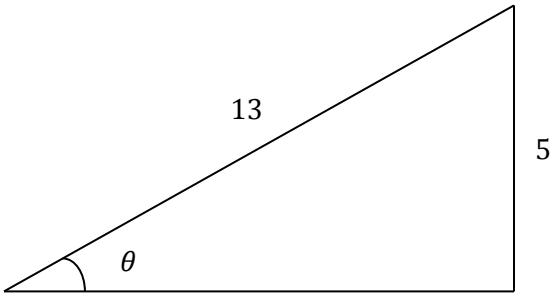
$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Problem 1. Find the exact value of the six trigonometric functions of the angle shown in the figure.



Problem 2. Sketch a right triangle corresponding to the trigonometric function of the acute angle θ.

$$\sin\theta = \frac{3}{8}$$

Problem 3. Let $\csc\theta = \frac{\sqrt{34}}{3}$, $\sec\theta = \frac{\sqrt{34}}{5}$. Find $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$.

Problem 4. Find the values of θ in degrees $(0 < \theta < 90°)$ and in radians $(0 < \theta < \pi/2)$ without a calculator.

a) $\tan\theta = \frac{\sqrt{3}}{3}$

b) $\sin\theta = \frac{1}{2}$

Problem 5. A biologist wants to know the width w of a river in order to properly set instruments for studying the pollutants in the water. From the point A, the biologist walks downstream 100 feet and sights to point C (which is directly across the river from point A). From the sighting at her new position, it is determined that the angle to point C is $\theta = 54°$. How wide is the river?

Problem 6. You are skiing down a mountain with a vertical height of 1500 feet. The distance from the top of the mountain to the base is 3000 feet. What is the angle of elevation from the base to the top of the mountain?

Problem 7. A steel cable zip-line is being constructed for a competition on a reality television show. One end of the zip-line is attached to a platform on top of a 140-foot pole. The other end of the zip-line is attached to the top of a 5-foot stake. The angle of elevation to the platform is $25°$. How long is the zip-line? How far is the stake from the pole?

Problem 8. Use trigonometric identities to transform the left side of the equation into the right side.

a) $\sin\theta \csc\theta = 1$

b) $\tan\alpha \cos\alpha = \sin\alpha$

c) $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$

d) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

Homework: Read section 4.3, do #1, 11, 15, 21, 23, 25, 33, 41, 49, 59, 63, 69, 72